Parameterized complexity : A tool to handle multiobjective problems

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Internship with the possibility to continue with a doctoral thesis.

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Scientific context
In a multi-objective problem, a solution S dominates a solution S' if for each objectif of the problem, the solution S performs better than the solution S'. This notion of domination determine a partial order $\leq$ on the set of the solutions. When solving a multi-objective problem, it is rare than there exists one solution S that dominates every other solution. The goal is then to find the pareto front that consists of all the solutions that are minimal with regard to the partial order $\leq$. There already exists several techniques in order to compute the pareto front, some of them can be found in [2].

A canonical problem studied in parameterized complexity is Vertex Cover that is known to be NP-hard (see [1] for instance). This problem can be solved in time $2^{O(n)} \cdot n^{O(1)}$ and cannot be solved in time $2^{o(n)} \cdot n^{O(1)}$, unless the exponential time hypothesis fails [1]. The main goal of parameterized complexity is to be able to catch the hardness of the problem within a parameter that can be a parameter like the size k of the solution. In particular, Vertex Cover can be solved in time $2^{O(k)} \cdot n^{O(1)}$ [1]. In these complexity results, we observe that the combinatorial explosion does not depend on n anymore but is caught by the parameter. More generally, if n is the size of the instance and x is a parameter, the main goal of the parameterized complexity is to find an FPT-algorithm, i.e., an algorithm with running time $f(x) \cdot n^{O(1)}$.

Goal of the internship
The parameterized complexity approach appears to be really efficient to handle single-objective parameterized problems. In this internship, we would like to extend the potential of parameterized complexity such that it can also handle multi-objective optimization problems

In particular, we want a parameterized complexity approach in order to handle Vertex Cover on a weighted graph where we want to minimize both the size of the solution and the weight of the solution at the same time.

This internship should then be organized as follows :

- Discover multi-objective optimization
- Discover parameterized complexity
- Appropriation of the Vertex Cover problem and the known techniques to solve it
- Appropriation of the Vertex Cover problem in the multi-objective optimization context
- Elaboration of an algorithm that solves Vertex Cover with regard to the two objectives
- Calcul of the complexity of the algorithm and improvement of it

Expected skills. Combinatorial optimization, Algorithms and complexity, Parameterized complexity or multi-objective optimization

References

[1] Marek Cygan, Fedor V. Fomin, Lukasz Kowalik, Daniel Lokshtanov, Dániel Marx, Marcin Pilipczuk, Michal Pilipczuk, and Saket Saurabh. *Parameterized Algorithms*. Springer, 2015.

[2] El-Ghazali Talbi. *Metaheuristics: From Design to Implementation*. Wiley, 2009.